Contents lists available at [ScienceDirect](http://www.sciencedirect.com/science/journal/09258388)



Journal of Alloys and Compounds journal homepage: [www.elsevier.com/locate/jallcom](http://www.elsevier.com/locate/jallcom)

# Theoretical study on the stability, elasticity, hardness and electronic structures of W–C binary compounds

# Yefei Li<sup>a,∗</sup>, Yimin Gao<sup>a,∗</sup>, Bing Xiao<sup>b</sup>, Ting Min<sup>a</sup>, Zijian Fan<sup>a</sup>, Shengqiang Ma<sup>a</sup>, Leilei Xu<sup>c</sup>

a State Key Laboratory for Mechanical Behavior of Materials, Xi'an Jiaotong University, Xi'an 710049, PR China

**b Department of Physics and Quantum Theory Group, School of Science and Engineering, Tulane University, New Orleans, LA 70118, USA** 

<sup>c</sup> Technology Center, Tianjin Pipe (Group) Corp Ltd, Tianjin 300301, PR China

# article info

Article history: Received 6 January 2010 Received in revised form 3 April 2010 Accepted 25 April 2010 Available online 5 May 2010

Keywords: Stability First-principles calculations Heat capacity Elastic constants Hardness

# ABSTRACT

The ground state properties of W–C binary compounds (h-WC, c-WC,  $\alpha$ -W $_2$ C,  $\beta$ -W $_2$ C,  $\gamma$ -W $_2$ C,  $\varepsilon$ -W $_2$ C) are studied in this paper by using first-principles calculations. Formation enthalpy and cohesive energy for each phase are calculated. The calculated elastic constants satisfy the Born–Huang's stability criterion, indicating all studied compounds are mechanically stable. All W–C compounds studied in this paper exhibit larger bulk modulus values than many other binary types of carbide such as Fe<sub>3</sub>C, Cr<sub>7</sub>C<sub>3</sub>, Cr<sub>3</sub>C, and TiC. Using a theoretical method based on the works of Šimůnek, the hardness of the crystal is estimated. The electronic structures of these compounds are calculated and discussed. Stoner's polarization theory for itinerant magnetism is applied to explain the observed paramagnetic behavior of the compounds. Moreover, the heat capacity is also calculated for each compound based on the knowledge of the elasticity and Debye temperature.

© 2010 Elsevier B.V. All rights reserved.

# **1. Introduction**

In recent decades the transition metal carbides are extensively investigated both by theoretical calculations and by experimental methods; because many of them exhibit outstanding mechanical properties and chemical stability, for instance they are very hard compounds and have high melting point, high thermal conductivity, etc. One of the most important compounds is the tungsten carbides: hexagonal and cubic mono-carbides WC, and four polymorphs  $(\alpha, \beta, \gamma, \varepsilon)$  of semi-carbide W<sub>2</sub>C. The properties of thermal stability and high elastic modulus determine the usage of W–C compounds in the production of wear resistant hard alloys, which form the basis of the metal cutting instruments, for example, the cemented carbide [\[1–4\]](#page-9-0) and high-speed steel [\[5–7\].](#page-9-0)

In all bulk W–C binary compounds, the investigations of hexagonal WC (h-WC) have been performed extensively, including the properties of bulk structures [8-10], the surfaces ((0001) [\[11,12\],](#page-9-0)  $(10\overline{1}0)$  [\[13\]](#page-9-0) and  $(11\overline{2}0)$  [\[13\],](#page-9-0) etc.) and the different interfaces (Al/WC  $[10]$ , Fe/WC  $[14]$ ). Liu et al.  $[8]$  investigated the structural and electronic properties of h-WC by performing a pseudopotential total-energy calculation. The obtained bulk modulus was 413 GPa which lies in the range of the measured values (329–577 GPa). They also found that the Fermi level locates in a deep minimum of the DOS, and the strong W–C bonds play an important role in the stability of WC. Zhukov and Gubanov [\[9\]](#page-9-0) confirmed the extraordinary high values of bulk modulus (655 GPa) and Debye temperature (648 K) of h-WC which explain in part the superior properties of WC as a cutting material. In addition to the hcp phase, WC also exists in the NaCl-type structure (c-WC). This is a high temperature phase and can be stabilized at room temperature by a rapid quenching process applied to the liquid state [\[15\].](#page-9-0) The c-WC is found to be superconducting with a transition temperature of 10.0 K [\[16\],](#page-9-0) and the h-WC is more stable than the c-WC. The electronic band structure and density of states show that the instability of NaCl-type structure is due to the occupation of anti-bonding states [\[17\]. W](#page-9-0)hile less information is available for the electronic structure, stability and mechanical property of W2C polymorphs. Until recently, Kurlov and Gusev [\[18\]](#page-9-0) summarized the phase equilibrium in the W–C binary system and the crystal structures of all W–C carbides, who pointed out that the hardness of WC decreases slightly with increasing temperature from 300 to 1200–1300 K. Suetin et al. [\[19,20\]](#page-9-0) reported that all W–C phases depending on their thermodynamic stability have the following sequence: h-WC> $\varepsilon$ -W<sub>2</sub>C> $\beta$ -W<sub>2</sub>C> $\gamma$ -W<sub>2</sub>C> $\alpha$ -W<sub>2</sub>C>c-WC. Meanwhile, the abrasive resistance of WC/W<sub>2</sub>C–(Co, Ni) alloys, the properties of cermets coatings and the WC/Metal matrix composites were investigated by other articles [\[21–23\]. I](#page-9-0)n this paper, our main interests are to study the chemical stability, electronic structures, mechanical properties (especially elastic modulus and hardness), and heat capacity of all W–C binary compounds.

<sup>∗</sup> Corresponding authors at: School of Materials Science and Engineering, Xi'an Jiaotong University, 28 Xianning West Road, Xi'an, Shaanxi Province 710049, PR China. Tel.: +86 29 82665479; fax: +86 29 82665479.

E-mail addresses: [yefeili@126.com](mailto:yefeili@126.com) (Y. Li), [ymgao@mail.xjtu.edu.cn](mailto:ymgao@mail.xjtu.edu.cn) (Y. Gao).

<sup>0925-8388/\$ –</sup> see front matter © 2010 Elsevier B.V. All rights reserved. doi:[10.1016/j.jallcom.2010.04.184](dx.doi.org/10.1016/j.jallcom.2010.04.184)

<span id="page-1-0"></span>

**Fig. 1.** Crystal structures calculated for (a) h-WC, (b) c-WC, (c)  $\alpha$ -W2C, (d)  $\beta$ -W2C, (e)  $\gamma$ -W2C.1, (f)  $\gamma$ -W2C.2 and (g)  $\varepsilon$ -W2C compounds, respectively. The small gray ball represents C atoms; the large blue ball refers to W atoms. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.).

#### **2. Methods and details**

Fig. 1 shows the crystal structures of W–C binary compounds. The details of the crystal structures of all W–C compounds can be found in the articles [\[18–20\].](#page-9-0) It is worth mentioning that in unit cell of  $\gamma$ -W<sub>2</sub>C, two tungsten atoms occupy the 2c positions and one carbon atom is randomly distributed between positions 2a with coordinates  $(000)$  and  $(001/2)$ ; In another words, the carbon atoms randomly occupy one-half of all octahedral interstices in the tungsten sublattice [\[18\]. T](#page-9-0)here-fore, in the papers [\[19,20\], S](#page-9-0)uetin considered the structure of  $\gamma$ -W<sub>2</sub>C as the uniform alternation of vacancies and carbon atoms along [0 0 1] crystal direction by using supercell method. Thus, in the present paper, we also used supercell method and allowed the uniform distribution of vacancies and carbon atoms along [0 0 1], [0 1 0] and [100] crystal directions. As a result, there are two crystal structures for  $\gamma$ -W<sub>2</sub>C, namely  $\gamma$ -W<sub>2</sub>C<sub>-1</sub> ([001] crystal direction) and  $\gamma$ -W<sub>2</sub>C<sub>-2</sub> ([010] or [100] crystal direction), because the symmetries of  $\gamma$ -W<sub>2</sub>C along [0 1 0] and [1 0 0] are same.

All calculations in this work were performed by first-principles calculation method based on density functional theory (DFT), as implemented in CASTEP code [\[24–26\]. T](#page-9-0)he ultra-soft pseudo potentials were employed to represent the interactions between ionic core and valence electrons. For W and C, the valence electrons considered are  $5s^25p^65d^46s^2$ ,  $2s^22p^2$ , respectively. In CASTEP code, the fully separable Kleinman–Bylander ultra-soft pseudopotentials were used and the relativistic effects for valence shells of heavy transition atoms were included using a scalar relativistic kinetic energy operator for Schrödinger like equation. Generalized gradient approximation (GGA) of PBE approach was used for exchange-correlation energy calculations [\[27\]. A](#page-9-0) kinetic energy cut-off value of 400 eV was used for plane wave expansions. The total-energy integrations were evaluated in the first irreducible Brillouin zone with Monkhorst–Pack scheme at special k points [\[28\], f](#page-9-0)or example  $10 \times 10 \times 10$  and  $15 \times 15 \times 15$  for electronic optimizations and elastic constants calculations, respectively. In order to investigate the chemical stability of these compounds at given P/T conditions, the cohesive energy and formation enthalpy were calculated using

$$
E_{\rm coh}(\rm WC) = E_{\rm tot}(\rm WC) - E_{\rm iso}(\rm W) - E_{\rm iso}(\rm C)
$$
\n(1)

$$
\Delta_r H(WC) = E_{coh}(WC) - E_{coh}(W) - E_{coh}(C)
$$
\n(2)

$$
E_{\rm coh}(W_2C) = E_{\rm tot}(W_2C) - 2E_{\rm iso}(W) - E_{\rm iso}(C)
$$
\n(3)

$$
\Delta_r H(W_2 C) = E_{\rm coh}(W_2 C) - 2E_{\rm coh}(W) - E_{\rm coh}(C)
$$
\n(4)

where  $E_{\text{coh}}(X)$  and  $\Delta_f H$  (X) is the cohesive energy, formation enthalpy of X phase, respectively;  $E_{\text{tot}}(X)$  is the total-energy of X material and  $E_{\text{iso}}(X)$  denotes the totalenergy of a single X atom. The influence of hydrostatic pressure on the formation enthalpy was investigated in this paper, the crystal structures were fully optimized at each pressure and then the total cell energy was obtained. For computational convenience, the calculations were performed at  $T = 0$  K, thus, the formation enthalpy equals to the free energy, which reflects the thermodynamical stability.

#### **3. Results and discussions**

#### 3.1. Stability

In Table 1, we depict the calculated cell parameters, cohesive energy and formation enthalpy of all modifications of WC and  $W<sub>2</sub>C$  compounds. Generally speaking, the calculated cell constants

#### **Table 1**

The calculated cell parameters (a, b, c, in Å;  $\rho$  in g/cm<sup>3</sup>; V<sub>cell</sub> in Å<sup>3</sup>), atomic positions for W and C atoms (fractional coordinates), cohesive energy (eV/atom), formation enthalpy (eV/atom) of W–C carbides (including h-, c-WC and  $\alpha$ -,  $\beta$ -,  $\gamma$ -,  $\varepsilon$ -W<sub>2</sub>C polymorphs).

Phase	h-WC	c-WC	$\alpha$ -W <sub>2</sub> C	$\beta$ -W <sub>2</sub> C	$\gamma$ -W <sub>2</sub> C <sub>-1</sub>	$\gamma$ -W <sub>2</sub> C <sub>-2</sub>	$\varepsilon$ -W <sub>2</sub> C
Space group P6m2		Fm3m	P3m1	Pbcn	$P_1$ <sup>1</sup>	$P_1$ <sup>1</sup>	P31m
a	2.906(2.906 <sup>a,b</sup> )		$4.351 (4.265a) 3.043 (3.001a)$	4.725 $(4.728a,c)$	3.043 (3.002 <sup>a</sup> , 2.992 <sup>d</sup> ) 3.029 (3.002 <sup>a</sup> )		5.211(5.184)
b				6.057 (6.009 a,c)	6.087	6.090	
	2.825(2.837 <sup>a,b</sup> )		4.655(4.728 <sup>a</sup> )	5.195 $(5.193^{a,c})$	$(4.707 (4.75^a, 4.725^d))$	$4.707(4.75^a)$	4.752(4.721 <sup>a</sup> )
$V_{\text{cell}}$	20.6558	82.3748	37.3251	148.6584	150.649	150.655	111.7365
$\rho$	15.74	15.79	16.89	16.97	16.742	16.741	16.93
W	(000)	(000)	$(0.3333 \ 0.6667 \ 0.25)$	(0.25 0.125 0.083)	(0.6667 0.3333 0.25)	(0.6667 0.3333 0.25)	(0.333300.25)
	$(0.3333 \ 0.6667 \ 0.5)$ $(0.5 \ 0.5 \ 0.5)$		(000)	(0.3750.25)	(000)	(000)	$(0 0 0)$ $(0.3333 0.6667 0.5)$
$E_{coh}$	$-10.64$	$-10.19$	$-10.79$	$-10.87$	$-10.89$	$-10.89$	$-10.86$
$\Delta_r H$	$-0.106(-0.170)$	0.341(0.281 <sup>e</sup> )	$0.065(0.063^e)$	$-0.018(-0.007e)$	$-0.029(0.006^e)$	$-0.030$	$-0.003$ ( $-0.013$ <sup>e</sup> )

<sup>a</sup> Experimental data summarized in Ref. [\[18\].](#page-9-0)

<sup>b</sup> Inorganic Crystal Structure Database (ICSD) #43380.

 $C$  ICSD #43017.

 $d$  ICSD #77567

<sup>e</sup> FP-LMTO method in Ref. [\[20\].](#page-9-0)

 $1\gamma$ -W<sub>2</sub>C has space group P6<sub>3</sub>/mmc, the supercell was used and the corresponding space group is P<sub>1</sub>.

of all W–C compounds are in good agreement with experimental results. The average deviation of our results to experimental results for lattice constants is 2%. Because GGA was used in our works; one could expect that it may overestimate the cell constants for some W–C binary compounds. Two thermodynamic parameters are used to determine the relative stability of all W–C phases. They are the cohesive energy and formation enthalpy. As defined in Eqs.  $(1)$ – $(4)$ , the smaller the negative values of these two parameters, the more stable the compound. The cohesive energies of all these carbides have negative values, but their formation enthalpies are quite different from each other. It is obvious that h-WC,  $\beta$ -W<sub>2</sub>C,  $\gamma$ -W<sub>2</sub>C<sub>-1</sub>,  $\gamma$ -W<sub>2</sub>C<sub>-2</sub> and  $\varepsilon$ -W<sub>2</sub>C are stable structures, which show negative enthalpy value of −0.106, −0.018, −0.029, −0.03 and −0.003 eV/atom., respectively. However, the formation enthalpy of c-WC and  $\alpha$ -W<sub>2</sub>C is positive, i.e. these species are meta- or un-stable, The stability sequence of six W–C phases forms the following order: h-WC >  $\gamma$ -W<sub>2</sub>C 2  $\approx \gamma$ -W<sub>2</sub>C 1 >  $\beta$ -W<sub>2</sub>C >  $\rm \varepsilon\text{-}W_2$ C >  $\rm \alpha\text{-}W_2$ C > c-WC, which differs to the results obtained by Suetin et al.[\[19,20\]. I](#page-9-0)t is probably caused by the different calculation methods such as potential functions mentioned above. Among four W<sub>2</sub>C polymorphs, two modified  $\gamma$ -W<sub>2</sub>C crystals are more stable, and the calculated cohesive energy and formation enthalpy of them are smaller than other three  $W<sub>2</sub>C$  crystals. The results may indicate that the randomly distributed carbon vacancies at (0 0 0 1) crystal plane do not show superior orientations. In addition, in the literature [\[18\], t](#page-9-0)he authors reported that the temperature induced consecutive phase transformations of  $W_2C$  polymorphs occur with the following sequences

$$
\gamma - W_2 C \rightleftharpoons \beta - W_2 C \tag{5}
$$

$$
\beta - W_2 C \rightleftharpoons \varepsilon - W_2 C \tag{6}
$$

$$
c - \text{WC} \rightleftharpoons h - \text{WC} \tag{7}
$$

The polymorphic transformation temperatures are ∼2750,  $\sim$ 2728, and  $\sim$ 3008 K, respectively, which were determined by using the high temperature differential thermal analysis involved in a highly sensitive optical system. Note that the Eq. (5) is a disorder–order transformation. These transformation sequences are in agreement with the formation enthalpy differences as can be seen from [Table 1.](#page-1-0) The large enthalpy difference between low and high temperature phases may be responsible for high transformation temperature.

On the other hand, the external pressure induced phase transformations are frequently observed in many oxides, carbides and borides of transition metals and which have been extensively studied. However, the data on the phase stability for the polymorphic modifications of the W–C binary compounds caused by hydrostatic pressure are absent. Fig. 2 presents the formation enthalpy curves as a function of hydrostatic pressures for all W–C structures. It is clear that the calculations for h- and c-WC polymorphs correctly predict the stability of h-WC is better than that of c-WC at high pressure, and the phase transformations are not driven by pressure, because the enthalpy curves almost parallel or departure from each other. For  $W_2C$  polymorphs, the  $\gamma$ -W<sub>2</sub>C phase is stabilized up to ~80 GPa and then transforms to  $\beta$ -W<sub>2</sub>C with the *Pbcn* structure, and  $\beta$ -W<sub>2</sub>C is found to be the most stable phase among W<sub>2</sub>C polymorphs at the pressure range of 80–200 GPa. Another pressure induced phase transformation occurs at 150 GPa between  $\gamma$ -W<sub>2</sub>C and  $\varepsilon$ -W<sub>2</sub>C. Calculating the phonon spectrum of these W<sub>2</sub>C polymorphs as a function of hydrostatic pressure can provide more details about the phase transformations which are not carried out in present paper. Comparing the enthalpy curves of WC in Fig. 2(a) to those of  $W_2C$  in Fig. 2(b), we conclude that the chemical stability of WC polymorphs is less sensitive to the hydrostatic pressure than that of  $W_2C$  compounds. Thus h-WC will have high mechanical



**Fig. 2.** Calculated formation enthalpy values vs hydrostatic pressures for the (a) hand c-WC polymorphs and (b)  $\alpha$ -,  $\beta$ -,  $\gamma$ -,  $\varepsilon$ -W<sub>2</sub>C polymorphs. For WC polymorphs, the  $\overline{P6}$ m2 structure is taken as reference; for W<sub>2</sub>C phases, enthalpies are shown with respect to the  $P\overline{3}m1$  structure.

stability at high pressures and is suitable to be used as reinforced phase in composites.

#### 3.2. Electronic structures

In this part, three parameters are used to indicate the electronic structures and chemical bonding characteristics of all W–C phases, which are density of states (DOS), electron density distribution map and population analysis. For h-WC, as shown in [Fig. 3a,](#page-3-0) the Fermi level is situated close to a minimum of the DOS qualitatively indicating its stability nature, while for c-WC, the Fermi level lies at the local maximum of DOS, destabilizing the NaCl-like structure of WC. In contrast to WC crystals, all  $W_2C$  polymorphs have relatively large values of DOS at Fermi level, which reproduce the results obtained by Suetin et al. [\[19,20\], i](#page-9-0)ndicating they are less stable than h-WC.

The calculated total electron density distribution maps of these compounds are shown in [Fig. 4.](#page-3-0) In all figures, even in the interstitial regions the electron density values are greatly larger than zero, which clearly indicate the metallic nature ofW–C compounds. The elongated contours along W–C bond axis can be ascribed to the covalent interactions; this provides an evidence to explain the covalent bonds caused by hybridization of C-2p and W-5d orbitals [\[9,10,19\].](#page-9-0) As was already mentioned, the calculated DOS of h-WC has a minimum near the Fermi level which is a reflection of the splitting of bonding and anti-bonding states created by the

<span id="page-3-0"></span>

Fig. 3. Total density of states of (a) WC and (b) W<sub>2</sub>C polymorphs. Dashed lines represent the Fermi level.

hybridization of 2p and 5d orbitals. The bonding states are mainly consisted of 2p bands of C and the anti-bonding states above the Fermi level are 5d bands of W. When  $W/C = 1$ , for instance h-WC, the bonding states below the Fermi level are fully occupied and the anti-bonding states are unoccupied. As a result, h-WC has the highest stability among W–C binary compounds. W<sub>2</sub>C represents the W rich condition, implying that the some anti-bonding states are also occupied. The calculated DOS curves for W–C binary compounds obviously have such features. There is a local minimum in the calculated DOS of  $\gamma$ -W<sub>2</sub>C<sub>-1</sub> and which is absent in  $\gamma$ -W<sub>2</sub>C<sub>-2</sub>. The calculated formation enthalpy for  $\gamma$ -W<sub>2</sub>C<sub>-1</sub> is comparable to  $\gamma$ -W<sub>2</sub>C<sub>-2</sub>.

Magnetism is rarely observed in compounds of 4d and 5d transition metals, and they are paramagnetic compounds. Stoner's polarization theory is prevalently used to explain the local magnetic moments of metal atoms for 3d, 4d and 5d transition metals and their compounds, because the DOS of them always has strong metallic character. Itinerant magnetism can be observed when Stoner's condition is satisfied, that is  $N_f$  > 1. Here,  $N_f$  is the density of d states only at the Fermi level in unit states/eV·atom and I characterizes the internal exchange parameter of d electrons (unit in eV/atom), I is independent of the crystal structures. It is previously suggested that for 3d, 4d and 5d transition metals, the I value is 0.9, 0.65 and 0.3, respectively [\[29–32\]. A](#page-9-0)s a result, the critical DOS values for 5d transition metals and the corresponding compounds are 3 states/eV·atom. As can be seen from [Table 2, n](#page-4-0)o W–C binary compound could fulfil such a condition, indicating that all of WC and W<sub>2</sub>C polymorphs are paramagnetic compounds. Furthermore, we also calculated Pauli paramagnetic susceptibilities for W–C binary compounds using  $\chi = \mu_B^2 D_F$ , where  $\chi$  is Pauli paramagnetic susceptibility,  $\mu_B$  is Bohr magnetic moment and  $D_f$  represents the total density of states at Fermi level, and the results are listed in [Table 2](#page-4-0) as well. The results for  $W_2C$  polymorphs are very close to ref. [\[19\].](#page-9-0)

Population analysis results can provide more insightful information on chemical bonding, which are listed in [Table 3. M](#page-4-0)ulliken method is applied for the overlap population and the charge calculations. We used Eqs. (8) and (9) to calculate the average bond length and the average overlap population:

$$
\overline{L}(AB) = \frac{\sum L_i N_i}{\sum N_i}
$$
 (8)

$$
\overline{n}(AB) = \frac{\sum n_i N_i}{\sum N_i}
$$
\n(9)

Here,  $\overline{L}(AB)$  and  $\overline{n}(AB)$  represent the average bond length and the mean bond population, respectively;  $N_i$  is the total number of i bond in the cell and  $L_i$  is the bond length of *i* type.

First of all, we want to list the electronegativity value of W and C atoms in Pauling scale: 2.55 (C), 1.7 (W). As shown in [Table 3, i](#page-4-0)t is clearly indicated that C carries the negative charges for all of the



**Fig. 4.** Total electron density distribution through (110), (001), (110), (010), (111), (111) and (1-10) slice intersecting both W and C atoms for (a) h-WC, (b) c-WC, (c) a-W2C, (d)  $\beta$ -W2C, (e)  $\gamma$ -W2C\_1, (f)  $\gamma$ -W2C\_2 and (g)  $\varepsilon$ -W2C compounds, respectively. The electron density contours were plotted from 0 to 2.0 e/Å<sup>3</sup>.

<span id="page-4-0"></span>**Table 2**



The calculated Stoner's parameters (N<sub>f</sub> I), Pauli paramagnetic susceptibilities ( $\chi$ , 10<sup>-4</sup> emu/mol) and the metallicity parameter for W–C binary compounds; D<sub>f</sub> is the total density of states at the Fermi level (states/(eV-atom)), and  $N_f$  is the DOS value at the Fermi level for d state only (states/(eV-atom)).

(a–c) The data are obtained using the linear muffin-tin orbital (LMTO) method in Refs. [\[17,34,35\], w](#page-9-0)hich differ from our results using partial wave analysis (PWA) method. (d) FLAPW-GGA method in Ref. [\[19,20\].](#page-9-0)

W–C compounds, and the value varies from −0.6 (h-WC) to −0.71  $(\epsilon-W_2C)$ . The largest positive charges are carried by W atoms of c-WC. One can propose two possible electron transfer paths intra or inter the C and W atoms. The one refers to p-d hybridized covalent bonding between C and W, and the other one is induced by the metallic bonding among W atoms. In the former case, the electrons are transferred from W to C, and can be designated as 6s (W) to 2p (C) for all W–C compounds. Besides, for  $\beta$ - and  $\varepsilon$ -W<sub>2</sub>C, W–W bonds show positive overlap populations, implying a metallic bonding character between tungsten atoms. From [Fig. 4](#page-3-0) it can be seen that the metallic W–W bond is so strong that it cannot be simply neglected. In fact, Stadler et al. [\[33\]](#page-9-0) studied the electronic structures of W<sub>2</sub>B both by using experimental methods and DFT calculations, they found that a shallow spitting of d bands, and which implies strong d-d metallic bonding between W atoms; for  $\alpha$ -W $_2$ C, W–W bonds have negative overlap populations, which indicates anti-bond states or strong electrostatic repulsion among tungsten atoms. The similar negative bond populations can also be seen in C–C bonds for  $\beta$ - and  $\gamma$ -W<sub>2</sub>C. Thus, the covalent interactions for all W–C compounds are mainly determined byW–C bonds. Otherwise, the metallicity of the tungsten carbides is ascribed to W–W bonds. The metallicity of the compound is estimated by

$$
f_m = \frac{n_m}{n_e} = \frac{k_B T D_f}{n_e} = \frac{0.026 D_f}{n_e}
$$
\n(10)

where  $D_f$  is the DOS value at the Fermi level in unit states/eV·cell, and T is the temperature;  $n_m$  and  $n_e$  are the thermal excited electrons and valence electron density of the cell, respectively;  $k_B$  is the Boltzmann constant.  $n_e$  is calculated by  $n_e = N/V_{cell}$ , N is the total number of valence electrons and  $V_{\text{cell}}$  is the cell volume. The related parameters and calculated results are shown in Table 2, from which we can observed that  $f_m$  increases in the following sequence: h-

## **Table 3**

Mulliken population analysis results of all W-C compounds:  $\overline{n}$  is average population,  $\overline{L}$  is average bonding length. Note that total charges of s and p orbitals of W are calculated as 5s + 6s and 5p + 6p, respectively.

Polymorph	Species (Ion)	$\mathsf S$	$\, {\bf p}$	$\mathbf d$	Total electrons	Charges
h-WC	$\mathsf C$ W	1.37 2.46 <sup>a</sup>	3.23 6.51 <sup>b</sup>	$\boldsymbol{0}$ 4.43	4.6 13.4	$-0.6$ $0.6\,$
$c$ -WC	$C(1-4)$ $W(1-4)$	1.39 2.35	$3.3\,$ 6.44	$\boldsymbol{0}$ 4.52	4.69 13.31	$-0.69$ 0.69
$\alpha$ -W <sub>2</sub> C	$\mathsf{C}$ $W(1-2)$	1.39 2.43	3.26 6.66	$\bf{0}$ 4.58	4.65 13.67	$-0.65$ 0.33
$\beta$ -W <sub>2</sub> C	$C(1-4)$ $W(1-8)$	1.38 2.41	3.32 6.66	$\boldsymbol{0}$ 4.59	4.69 13.65	$-0.69$ 0.35
$\gamma$ -W <sub>2</sub> C <sub>-1</sub>	$C(1-4)$ $W(1-8)$	1.38 2.40	3.31 6.67	4.58	4.69 13.65	$-0.69$ 0.35
$\gamma$ -W <sub>2</sub> C <sub>-2</sub>	$C(1-4)$ $W(1-8)$	1.38 2.40	3.31 6.67	4.58	4.69 13.65	$-0.69$ 0.35
$\varepsilon$ -W <sub>2</sub> C	C(1) $C(2-3)$ $W(1-6)$	1.37 1.38 2.41	3.34 3.31 6.66	$\boldsymbol{0}$ $\bf{0}$ 4.58	4.71 4.69 13.65	$-0.71$ $-0.69$ 0.35
Polymorph	Bond	$\overline{n}$ (electrons)		$\overline{L}(\mathring{A})$		
h-WC $c-WC$	$C-W$ $C-W$	2.1 $\rm 0.8$		2.19 2.18		
$\alpha$ -W <sub>2</sub> C	$C-W$ $W-W$	0.89 $-0.20$		2.12 2.89		
$\beta$ -W <sub>2</sub> C	$C-W$ $W-W$ $C-C$ <b>WW</b>	0.31 0.13 $\!-0.08$ 0.27		2.11 2.91 2.99 2.99		
$\gamma$ -W <sub>2</sub> C <sub>-1</sub>	$C-W$ $W-W$	0.34 0.23		2.12 2.91		
$\gamma$ -W <sub>2</sub> C <sub>-2</sub>	$C-W$ $\mathsf{W}\text{-}\mathsf{W}$	0.47 0.14		2.12 2.91		
$\varepsilon$ -W <sub>2</sub> C	$C-W$ $W-W$	$0.26\,$ 0.16		2.11 2.96		

Note: (a)  $5s + 6s$ ; (b)  $5p + 6p$ .

<span id="page-5-0"></span>Calculated values for the independent elastic constants (C<sub>ij</sub>, in GPa), bulk moduli (B, in GPa), shear moduli (G, in GPa), Young modulus (E, in GPa) and Poisson's ratio (v) of W–C binary compounds.



<sup>a</sup> Calculated by FLAPW with GGA scheme in Ref. [\[41\].](#page-9-0)

b FLAPW-GGA calculation results in Ref. [\[42\].](#page-9-0)

 $\epsilon$  Experimental results measured by the high-frequency ultrasonic method in Ref. [\[43\].](#page-9-0)

<sup>d</sup> FLAPW-GGA calculation results in Ref. [\[44\].](#page-9-0)

e Calculation data from Ref. [\[45\],](#page-9-0) at the temperature of 50 K.

 $f$  Ref. [\[46\].](#page-9-0)

WC <  $\alpha$ -W<sub>2</sub>C <  $\varepsilon$ -W<sub>2</sub>C < c-WC <  $\beta$ -W<sub>2</sub>C <  $\gamma$ -W<sub>2</sub>C <sup>2</sup> <  $\gamma$ -W<sub>2</sub>C <sup>1</sup>. Thus, the maximal and minimal "metallicity" correspond to the  $\gamma$ -W<sub>2</sub>C and the h-WC, respectively.

Based on the above discussions, the bonding behaviors of W–C compounds are the combinations of metallic, covalent and ionic bonds, which lead to the high melting point, high hardness, and good electric conductivity. In fact, transition metals form many sorts of carbides, such as TiC, VC, Fe<sub>3</sub>C, and Cr<sub>7</sub>C<sub>3</sub>, in which the mixture bonding states are always observed [\[36,37\].](#page-9-0)

# 3.3. Mechanical properties

The mechanical performances of the studied W–C system compounds play an important role for the application as wear resistance material. The elastic stiffness determines the response of a crystal to an imposed strain [\[38\].](#page-9-0) It is essential to investigate the elastic stiffness to understand the mechanical properties of W–C compounds. Therefore, a stress–strain approach, based on the generalized Hooke's law, was employed to calculate elastic constants in this work. The calculated independent elastic stiffness constants  $(C_{ii})$  are illustrated in [Table 4.](#page-5-0) According to Born–Huang's lattice dynamical theory, the mechanical stability criterions can be expressed as [\[39,40\]:](#page-9-0)

Hexagonal system (for h-WC,  $\gamma$ -W<sub>2</sub>C<sub>-1</sub> and  $\gamma$ -W<sub>2</sub>C<sub>-2</sub>):

$$
C_{11} > 0, C_{44} > 0, (C_{11} - C_{12}) > 0, (C_{11} + C_{12})C_{33} > 2C_{13}^2
$$
 (11)

Cubic system (for c-WC):

$$
C_{11} > 0, \ \ C_{44} > 0, \ \ C_{11} - C_{12} > 0, \ \ C_{11} + 2C_{12} > 0 \tag{12}
$$

Trigonal system (for  $\alpha$ -W $_2$ C and  $\varepsilon$ -W $_2$ C):

$$
C_{11} > |C_{12}|, (C_{11} + C_{12})C_{33} > 2C_{13}^2, (C_{11} - C_{12})C_{44} - 2C_{14}^2 > 0
$$
\n(13)

Orthorhombic system (for  $\beta$ -W<sub>2</sub>C):

$$
C_{11} + C_{12} + C_{33} + 2C_{12} + 2C_{13} + 2C_{23} > 0, \quad C_{11} + C_{22} > 2C_{12}, \quad C_{22}
$$

$$
+ C_{33} > 2C_{23}, \quad C_{11} + C_{33} > 2C_{13}, \quad C_{ii} > 0 \quad (i = 1 - 6) \quad (14)
$$

It is worth noting that the crystal structures of  $\gamma$ -W<sub>2</sub>C<sub>-1</sub> and  $\gamma$ -W<sub>2</sub>C<sub>-2</sub> should have hexagonal symmetry (with space groups of  $P6<sub>3</sub>/mmc$ ), since the supercell method is carried out to deal with  $\gamma$ - $W<sub>2</sub>C$  (with space group  $P<sub>1</sub>$ ). Therefore, only the independent elastic constants identical to hexagonal cell are listed in [Table 4](#page-5-0) for two  $\gamma$ -W<sub>2</sub>C modifications. As shown in [Table 4,](#page-5-0) the calculated elastic constants of each W–C compound satisfy the above corresponding criterion, respectively, indicating that all of W–C compounds are mechanically stable. For W–C system compounds, the calculated  $C_{11}$  and  $C_{33}$  are very large among elastic constants which indicates that they are very incompressible under uniaxial stress along  $x$  ( $\varepsilon_{11}$ ) or  $z$  ( $\varepsilon_{33}$ ) axis. Besides, for two WC polymorphs, the obtained  $C_{11}$  and  $C_{33}$  of h-WC are larger than c-WC, indicating the strong directional bonding in h-WC crystal along [1 0 0] direction. Among all W–C binary compounds, h-WC has the largest  $C_{11}$  value as 711.6 GPa. The calculated elastic constants of h-WC are in agreement with other references. For c-WC, the agreement with FLAPW results varies for different elastic constants. The largest deviation between them is 30 GPa.  $C_{44}$  indicates the resistances of the crystal with respect to the shear strain at (1 0 0) plane, and which is closely related to the shear modulus. From [Table 4, o](#page-5-0)ne can see that h-WC has the largest  $C_{44}$  value for all studied W–C binary compounds and  $\gamma$ -W<sub>2</sub>C<sub>-2</sub> has the largest C<sub>44</sub> value among W<sub>2</sub>C polymorphs. We also calculated  $C_{11}/C_{33}$ ,  $C_{12}/C_{13}$  and  $C_{44}/C_{66}$  values in order to illustrate the elastic anisotropy for eachW–C compound. Due to the symmetric restrictions on tensor elements, the obtained values are unity for c-WC. However, for other W–C binary compounds, the calculated ratios deviate from the unity, showing the mechanical anisotropy. The bulk modulus  $(B)$ , shear modulus  $(G)$ , Young's modulus  $(E)$  and Poisson's ratio ( $v$ ) for polycrystalline crystal can be obtained from independent single crystal elastic constants. Usually, the polycrystalline modulus is estimated within Voigt and Reuss methods [\[38\],](#page-9-0) in which the former method is based on assumption of uniform strain throughout a polycrystal, which is given by:

$$
B_V = \frac{1}{9}(C_{11} + C_{22} + C_{33}) + \frac{2}{9}(C_{12} + C_{23} + C_{13})
$$
\n(15)

$$
G_V = \frac{1}{15}(C_{11} + C_{22} + C_{33}) - \frac{1}{15}(C_{12} + C_{23} + C_{13}) + \frac{1}{5}(C_{44} + C_{55} + C_{66})
$$
\n(16)

The latter method assumes a uniform stress and gives B and G as functions of the elastic compliance constants  $S_{ij}$ , which is the inverse matrix of  $C_{ij}$ .

$$
\frac{1}{B_{\rm R}} = (S_{11} + S_{22} + S_{33}) + 2(S_{12} + S_{23} + S_{13})
$$
\n(17)

$$
\frac{1}{G_R} = \frac{4}{15}(S_{11} + S_{22} + S_{33}) - \frac{4}{15}(S_{12} + S_{23} + S_{13})
$$

$$
+ \frac{1}{5}(S_{44} + S_{55} + S_{66})
$$
(18)

where the subscripts V and R indicate the Voigt and Reuss averages. The arithmetic average of Voigt and Reuss bounds is termed as the Voigt–Reuss–Hill approximations, and it is considered as the best estimation of the theoretical polycrystalline elastic modulus. In addition, the polycrystalline Young's modulus and Poisson's ratio can be calculated from the values of elastic modulus as follows [\[40\]:](#page-9-0)

$$
E = \frac{9BG}{3B + G} \tag{19}
$$

$$
v = \frac{3B - 2G}{2(3B + G)}
$$
 (20)

Bulk modulus reflects the compressibility of the solid under hydrostatic pressure. The bulk modulus of W–C system compounds are 393, 375.9, 318.2, 340.6, 327.5, 327.9 and 339.8 GPa for h-WC, c-WC, α-W<sub>2</sub>C, β-W<sub>2</sub>C,  $\gamma$ -W<sub>2</sub>C<sub>-</sub>1,  $\gamma$ -W<sub>2</sub>C<sub>-</sub>2 and  $\varepsilon$ -W<sub>2</sub>C phases, respectively, which exhibit larger values than the common carbides such as Fe<sub>3</sub>C (276 [\[47\], 2](#page-9-0)26.8 [\[48\]\),](#page-9-0) Cr<sub>7</sub>C<sub>3</sub> (311.7 [\[37\], 3](#page-9-0)09 [\[49\]\),](#page-9-0) Cr<sub>3</sub>C (298.24 [\[37\]\),](#page-9-0) and TiC (249 [\[50\], 2](#page-9-0)42 [\[51\]\),](#page-9-0) but smaller than that of diamond (436.8 [\[52\]\).](#page-9-0) The Poisson's ratios ( $v$ ) of there compounds range from 0.207 (h-WC) to 0.313 (c-WC), which is near 0.3 and clearly illustrates the strong metallic character of W–C compounds. The largest discrepancy of  $v$  from 0.3 is observed for h-WC mainly caused by the ionic nature of crystal (typically, for ionic crystal, the value is near 0.25). Then the shear modulus is calculated and the values are 286.2, 160.7, 158.8, 172.6, 158.4, 174.7 and 173.8 GPa for h-, c-WC and all kinds of W<sub>2</sub>C polymorphs. The ratio of  $B/G$  is frequently used to indicate the ductility of the compound, it is supposed that for the brittle compound, B/G is smaller than 1.75 (for diamond  $B/G = 0.8$ ) and for metallic compound  $B/G$  is greater than 1.75 (for Al  $B/G = 2.74$ ). In our case, the calculated results clearly imply that h-WC is more brittle than other W–C compounds, since the Fermi level is situated close to a minimum of the DOS and the metallicity in h-WC is less remarkable than other W–C phases. Finally, the Debye temperature for each compound is calculated based on the knowledge of elastic constants and will be discussed in Section [3.4.](#page-8-0)





<sup>a</sup> Ref. [\[35\].](#page-9-0) <sup>b</sup> Ref. [\[52\].](#page-9-0)

 $c$  Ref. [\[53\].](#page-9-0)



**Fig. 5.** The compressibility curves of W–C binary compounds; for comparisons, the profiles of Fe3C, Cu and diamond are also shown.

In this part, we would like to discuss the compressibility of all W-C compounds. We fitted the total-energy  $(E)$  as function of cell volume (V) by using well-known Birch–Murnaghan equation of state (Eq. (21)).

$$
\begin{cases}\nE(V) = \frac{B_0 V_0}{B'_0} \left[ \frac{1}{B'_0 - 1} \left( \frac{V}{V_0} \right)^{(1 - B'_0)} + \frac{V}{V_0} + \frac{B'_0}{1 - B'_0} \right] + E_0 \\
P(V) = \frac{B_0}{B'_0} \left[ \left( \frac{V_0}{V} \right)^{(B'_0)} - 1 \right]\n\end{cases} \tag{21}
$$

Here, because the applied stress is up to 200 GPa, and the crystal cells have been compressed into only 3/4 of the equilibrium cell volumes for all W–C compounds, which may cause large errors because of the plastic deformation. Thus we fitted the E–V curves again by changing the cell volumes at the range of  $-10$  to 10%, from which we obtained several mechanical parameters as shown in Table 5. The obtained bulk moduli of all W–C compounds are close to the results in [Table 4, w](#page-5-0)hich indicate the reliability of our calculations. Fig. 5 shows the compressibility of W–C compounds. For comparisons, the results for Cu and diamond are also shown. The hardest material always situated in the upper part of the figure, because it is hard to compress, the relationship between the hardness and bulk modulus is still unclear, but hard materials usually favor large bulk modulus. In summary, W–C binary compounds are more incompressible than  $Cr_7C_3$ , Fe<sub>3</sub>C, Cr<sub>3</sub>C, TiC, etc.

The hardness of a material always plays an important role in its applications, especially using as an abrasive resistant phase [\[54\].](#page-9-0) Thus it is necessary to investigate the hardness of W–C carbides because of its foundation status in the wear resistance materials. According to Šimůnek's theory [\[55,56\], t](#page-9-0)he theoretical hardness of single crystal can be calculated within the following forms.

$$
H = \frac{C}{\Omega} n \left[ \prod_{i,j=1}^{n} b_{ij} s_{ij} \right]^{1/n} e^{-\sigma f_e}
$$
 (22)

$$
f_e = 1 - \left\lceil k \frac{\left(\prod_{i=1}^k e_i\right)^{1/k}}{\sum_{i=1}^k e_i} \right\rceil^2 \tag{23}
$$

$$
s_{ij} = \frac{\sqrt{e_i e_j}}{n_i n_j d_{ij}}\tag{24}
$$

where the reference energy  $e_i = Z_i/R_i, Z_i$  is the valence electron number of the atom *i*, and  $R_i$  is the radius of the sphere in which  $Z_i$ electrons are contained, because the change of  $e_i$  values for same atom in difference compounds are small [\[55\], s](#page-9-0)o in this work, we assumed that  $e_C$  and  $e_W$  are 3.764 and 3.971, respectively;  $n_i$  and  $n_i$  are coordination numbers of atoms *i* and *j*, respectively;  $d_{ii}$  is the interatomic distance between atoms  $i$  and  $j$ ;  $k$  corresponds to the number of different atoms in the system;  $s_{ij}$  is the strength of the individual bond between atoms *i* and *j*;  $b_{ij}$  is the individual bond in the unit cell; the constants C = 1550 and  $\sigma$  = 4 [\[55\]](#page-9-0) are used throughout this work. Using Eqs.(22)–(24) we have calculated the hardness of all W–C binary compounds as shown in Table 6.

The obtained hardness of h-WC is in satisfactory agreement with the previous calculation by Simunek and Vackář [\[55\].](#page-9-0) It is interesting that B and G of h-WC are as high as 393 and 286.2 GPa, respectively, but its hardness is only 22 GPa (for diamond, the hardness is 95.4 GPa [\[55\]\),](#page-9-0) providing a direct evidence that it is difficult to describe hardness quantitatively only by elastic moduli of B and G. Moreover, it is noteworthy that the hardness values for h-WC and c-WC are all smaller than those of  $W<sub>2</sub>C$  polymorphs. This is mainly

**Table 6**

The predicted hardness and parameters related to the hardness calculations of h-, c-WC and  $\alpha$ -,  $\beta$ -,  $\gamma$ -,  $\varepsilon$ -W $_2$ C polymorphs.

Phase	h-WC	$c-WC$	$\alpha$ -W <sub>2</sub> C	$B-W_2C$	$v-W_2C_1$	$v-W_2C_2$	$\varepsilon$ -W <sub>2</sub> C
$d_{12}$	2.193(2.194)	2.176	2.116	2.110	2.121	2.121	2.103
n <sub>1</sub>			6		6		
n <sub>2</sub>							
$b_{12}$		24		24	24	24	18
$s_{ii}$ (W-C)	0.0490	0.0494	0.1015	0.1018	0.1013	0.1013	0.1021
H(GPa)	22 (21.5 <sup>a</sup> 30 <sup>b</sup> , 35.6 <sup>c</sup> , 26.4 <sup>d</sup> )	22.25	25.22	25.4	24.94	24.94	25.42

<sup>a</sup> Ref. [\[55\].](#page-9-0)

<sup>b</sup> Ref. [\[57\].](#page-9-0)

<sup>c</sup> Ref. [\[58\].](#page-9-0)

 $d$  Ref. [\[59\].](#page-9-0)

# <span id="page-8-0"></span>**Table 7**

Theoretically calculated thermal properties of the W–C compounds, including the longitudinal sound velocity  $(v_1, m/s)$ , shear sound velocity  $(v_5, m/s)$ , and average sound velocity ( $v_m$ , m/s), Debye temperature ( $\Theta_{\rm D}$ , K), the characteristic parameters of electron ( $\gamma$ , mJ/(K<sup>2</sup>·mol)) and phonon ( $\beta$ , mJ/(K<sup>4</sup>·mol)) specific heat.



a The values are derived by measuring the low-temperature heat capacities of h-WC from 1.5 to 15 K in Ref. [\[64\].](#page-9-0)

**b** Theoretical data in Ref. [\[19\].](#page-9-0)

<sup>c</sup> Ref. [\[17\].](#page-9-0)

<sup>d</sup> Theoretical data in Ref. [\[20\].](#page-9-0)

due to the small coordination numbers of W atoms for  $W_2C$  polymorphs. For all  $W<sub>2</sub>C$  structures, W atoms are placed in the corner of an irregular tetrahedron which result in the large bond strength  $S_{ii}$ .

#### 3.4. Heat capacity

Once known the elastic constants and electronic structures of the compound, one can calculate the Debye temperature and heat capacity at the low-temperature. Debye temperature gives some insight into the thermodynamics of material from the elastic properties; it can be used to distinguish between high- and lowtemperature regions for a solid. For  $T > \Theta_D$ , all modes have energy of  $k_{\text{B}}T$ ; and for  $T < \Theta_D$  one expects high-frequency modes to be frozen [\[60\]. T](#page-9-0)he  $\Theta_{\rm D}$  can be estimated from the average sound velocity by the following equations [\[61,62\]:](#page-9-0)

$$
\Theta_{\rm D} = \frac{h}{k_{\rm B}} \left[ \frac{3n}{4\pi} \left( \frac{N_{\rm A}\rho}{M} \right) \right]^{1/3} \nu_m \tag{25}
$$

$$
v_m = \left[\frac{1}{3}\left(\frac{2}{v_s^3} + \frac{1}{v_l^3}\right)\right]^{-1/3} \tag{26}
$$

$$
v_1 = \sqrt{\left(B + \frac{4}{3}G\right)\frac{1}{\rho}}, \quad v_s = \sqrt{G/\rho} \tag{27}
$$

where  $\Theta_{\rm D}$  is the Debye temperature; h and  $k_{\rm B}$  are the Planck and Boltzmann constant, respectively;  $n$  is the total number of atoms per formula;  $N_A$  represents the Avogadro number;  $M$  is the molecular weight per formula;  $v_m$  is the average sound velocity;  $v_1$  is the longitudinal sound velocity and  $v_s$  the shear sound velocity. The calculated results of  $\Theta_{\text{D}}$ ,  $v_1$  and  $v_s$  are listed in Table 7, the calculated averaged elastic wave velocities of these carbides are relatively large which are around 3500 m/s. A reasonable explanation is these compounds have large B and G values. The largest  $\Theta_{\rm D}$ is 591.32 K for h-WC. It is known that the  $\Theta_{\rm D}$  can be used to characterize the strength of covalent bonds in solids. From Table 7, we can conclude that the covalent bond in h-WC is stronger than the other carbides. Besides, the sequence of  $\Theta_{\rm D}$  for W–C compounds is: h-WC> $\gamma$ -W<sub>2</sub>C\_2  $\approx$  e-W<sub>2</sub>C  $\approx$   $\beta$ -W<sub>2</sub>C >  $\alpha$ -W<sub>2</sub>C > c-WC >  $\gamma$ -W<sub>2</sub>C\_1, which is consistent with the observed trend of Young's modulus and shear modulus as shown in [Table 4](#page-5-0) Combining the values of  $\Theta_{\text{D}}$ and the Young's modulus, we may conclude that the mechanical stability and thermal stability of h-WC is the best one among all W–C compounds.

Besides, since the tungsten carbides studied in the present paper have metallic feature at the Fermi level, the calculations for the electronic structures and elastic constants enable us to estimate the heat capacity  $(c_n)$  at the low-temperature, which is given as

$$
c_p(T) = \gamma T + \beta T^3 \tag{28}
$$

$$
\gamma = \frac{1}{3}\pi^2 k_B^2 D_f \tag{29}
$$

$$
\beta = \frac{12\pi^4 R n}{5\Theta_{\text{D}}^3} \tag{30}
$$

In the equations,  $\gamma$  and  $\beta$  are the coefficients of electronic and lattice heat capacities, respectively; n is the total number of atoms per formula unit and R is the molar gas constant. Heat capacity is an invaluable tool to explore the fundamental properties of materials. Note that  $\Theta_{\text{D}}$ , as a rule, only describes the temperature dependence of  $c_p$  for  $T < \Theta_D/10$  [\[63\]. T](#page-9-0)herefore, In Fig. 6, we plot  $c_p$  versus T in the 0–50 K temperature range for all W–C compounds. At temperatures up to 9 K, as can be seen from Fig. 6, the main contribution to  $c_p$  is the excitation of electrons, and the values of  $c_p$  have the following sequence: h-WC < c-WC <  $\gamma$ - $W_2C_2 \approx \beta$ -W<sub>2</sub>C < α-W<sub>2</sub>C < ε-W<sub>2s</sub>C<  $\gamma$ -W<sub>2</sub>C<sub>-</sub>1, which are as same as the sequences of  $\gamma$  indicating the heat capacities are determined by electrons at first; besides, in Table 7, the smallest value of  $\gamma$  = 0.36 mJ/(K<sup>2</sup>·mol) is attributed to h-WC, because it has the smallest DOS value at the Fermi level, so h-WC has weakest metallic nature among all W–C compounds. In the range of 9–50 K, contributions from phonon excitations must be taken into account, as a result, the growth trend of  $c_p$  is: h-WC < c-WC <  $\gamma$ -W<sub>2</sub>C\_2  $\approx$   $\beta$ -W<sub>2</sub>C <  $\varepsilon$ -W<sub>2</sub>C <  $\alpha$ -W<sub>2</sub>C <  $\gamma$ -W<sub>2</sub>C  $\stackrel{.}{.1}$ . Thus, we can conclude that the heat capacity from electrons dominates the  $c_p$  curve at first, when T further raises the heat capacity is determined by phonon excitations. In addition, in the literature [\[45\], t](#page-9-0)he authors reported the values of  $c_p$  for h-WC in a large temperature range of 50–500 K, when  $T = 50$  K the value of 3.12 J/(K·mol) was achieved. Note that at  $T = 50$  K the agreement between the work and the calculated



**Fig. 6.** Heat capacity of W–C compounds plotted in the range of 0–50 K.

<span id="page-9-0"></span>results  $(2.39$  J/(K·mol)) in our paper is pretty good. When  $T > 50$  K, the variations of  $c_p$  follow the well-known rule, namely Debye's quasi-harmonic approximation. Unfortunately, similar experimental  $c_p$  data for other five W–C compounds are not available at the moment. This paper may be useful for low-temperature heat capacity research of W–C system compounds.

# **4. Conclusions**

We used first-principles calculations to study the properties of W–C compounds. The Fermi level of h-WC is situated close to a minimum of the DOS that qualitatively indicates its stability nature; for c-WC, the Fermi level lies at the local maximum of DOS destabilizes the NaCl-like structure of WC, while all  $W<sub>2</sub>C$  polymorphs have relatively large DOS values at the Fermi level, indicating they are less stable than h-WC. The calculated formation enthalpy values indicate that h-WC,  $\beta$ -W<sub>2</sub>C,  $\gamma$ -W<sub>2</sub>C<sub>-1</sub>,  $\gamma$ -W<sub>2</sub>C<sub>-2</sub> and  $\varepsilon$ -W<sub>2</sub>C are thermodynamically stable, while c-WC and  $\alpha$ -W<sub>2</sub>C appear to be metastable or unstable. The calculated total electron density distributionmaps clearly deduce themetallic nature ofW–C compounds; since in all of these maps, even in the interstitial regions the electron density values are larger than zero. The elongated contours along W–C bond axis can be ascribed to the covalent interactions. From the population results, it is found that the charge transfer from W to C is significant. Therefore, the bonding behaviors of W–C compounds are combinations of covalent, metallic, and ionic nature. Based on the Stoner's polarization theory for itinerant magnetism, all of W–C binary compounds are in paramagnetic state. Moreover, we also calculated Pauli paramagnetic susceptibilities of them, and the results are in agreement with the available theoretical values.

The bulk modulus and the first pressure derivative are estimated by fitting the EOS for each compound. The bulk modulus of W–C system compounds is 400.9, 374.4, 343.3, 353.5, 343.9, 343 and 341.9GPa for h-WC, c-WC,  $\alpha$ -W<sub>2</sub>C,  $\beta$ -W<sub>2</sub>C,  $\gamma$ -W<sub>2</sub>C and  $\varepsilon$ -W<sub>2</sub>C, respectively, which exhibit larger values than the common carbides such as  $Cr_7C_3$ , Fe<sub>3</sub>C, Cr<sub>3</sub>C, and TiC; All W–C binary compounds are more incompressible than  $Cr_7C_3$ , Fe<sub>3</sub>C, Cr<sub>3</sub>C, TiC, etc. The full set of elastic constants is calculated for each compound based on strain–stress method using CASTEP code. The calculated elastic constants satisfy the Born–Huang's stability criterion, indicating that all these compounds aremechanically stable. The largest bulk modulus, shear modulus, and Young modulus are attributed to h-WC and the value is 393, 286.2, and 690.9 GPa, respectively. The calculated hardness values of h-WC and c-WC are smaller than that of  $W<sub>2</sub>C$  polymorphs; this is mainly due to the less coordination numbers of atom W for  $\alpha$ -W<sub>2</sub>C,  $\beta$ -W<sub>2</sub>C,  $\gamma$ -W<sub>2</sub>C and  $\varepsilon$ -W<sub>2</sub>C. The Debye temperature and heat capacity for these compounds are also calculated, the agreement between our results and other references is pretty good.

## **Acknowledgments**

The authors appreciate Dr. X.J. Xie for useful discussion and thank to Prof. Y.H. Chen for providing a SGI working station and the CASTEP code. This research is supported by the Natural Science Foundation of China (no. 50872109), the 863 project in China (no. 2009AA03Z524), theCooperation Foundation for Industry, University and Research Institute, Guangdong Province and Ministry of Education of China (no. 2008B090500242), and the Economic and Trade Commission Creative Technology Program, Guangdong Province of China (no. 200872215).

## **References**

[1] Y. Torres, M. Anglada, L. Llanes, Int. J. Refract. Met. Hard Mater. 19 (2001) 341.

- [2] U. Beste, T. Hartzell, H. Engqvist, N. Axén, Wear 249 (2001) 324.
- [3] C. Karatas, B.S. Yilbas, A. Aleem, M. Ahsan, J. Mater. Process. Technol. 183 (2007) 234.
- [4] K. Bonny, P. De Baets, W. Ost, S. Huang, J. Vleugels, W. Liu, B. Lauwer, Int. J. Refract. Met. Hard Mater. 27 (2009) 350.
- [5] A.S. Chaus, M. Hudáková, Wear 267 (2009) 1051.
- [6] M. Pellizzari, D. Cescato, M.G. De Flora, Wear 267 (2009) 467.
- [7] M. Wiefssner, M. Leisch, H. Emminger, A. Kulmburg, Mater. Charact. 59 (2008) 937.
- [8] A.Y. Liu, R.M. Wentzcovitch, M.L. Cohen, Phys. Rev. B 38 (1988) 9483.
- V.P. Zhukov, V.A. Gubanov, Solid State Commun. 56 (1985) 51.
- [10] D.J. Siegel, L.G. Hector, J.B. Adams, Surf. Sci. 498 (2002) 321.
- [11] C.G. Larsson, J.B. Pendry, L.I. Johansson, Surf. Sci. 162 (1985) 19.
- [12] P.M. Stefan, M.L. Shek, W.E. Spicer, Surf. Sci. 149 (1985) 423.
- [13] F. Marinelli, A. Jelea, A. Allouche, Surf. Sci. 601 (2007) 578.
- [14] D. Lou, J. Hellman, D. Luhulima, J. Liimatainen, V.K. Lindroos, Mater. Sci. Eng. A 340 (2003) 155.
- [15] A. Antoni-Zdziobek, J.Y. Shen, M. Durand-Charre, Int. J. Refract. Met. Hard Mater. 26 (2008) 372.
- [16] R.H. Willens, E. Buchler, Appl. Phys. Lett. 7 (1967) 25.
- [17] M. Rajagopalan, P. Saigeetha, G. Kalpana, B. Palanivel, Jpn. J. Appl. Phys. 33 (1994) 1847.
- [18] A.S. Kurlov, A.I. Gusev, Russ. Chem. Rev. 75 (2006) 617.
- [19] D.V. Suetin, I.R. Shein, A.L. Ivanovskii, J. Phys. Chem. Solids 70 (2009) 64.
- D.V. Suetin, I.R. Shein, A.S. Kurlov, A.I. Gusev, A.L. Ivanovskii, Phys. Solid State 50 (2008) 1420.
- [21] A. Klimpel, L.A. Dobrzański, A. Lisiecki, D. Janicki, J. Mater. Process. Technol. 164 (2005) 1068.
- [22] T. Li, Q.F. Li, J.Y.H. Fuh, P.C. Yu, C.C. Wu, Mater. Sci. Eng. A 430 (2006) 113.
- [23] M.F. Morks, Y. Gao, N.F. Fahim, F.U. Yinquing, Mater. Lett. 60 (2006) 1049.
- [24] M.D. Segall, J.D. Philip, M.J. Lindan, C. Probert, J. Pickard, P.J. Hasnip, S.J. Clark, M.C. Payne, J. Phys.: Condens. Matter 14 (2002) 2717.
- [25] A.E. Mattsson, P.A. Schultz, M.P. Desjarlais, T.R. Mattsson, K. Leung, Modell. Simul. Mater. Sci. Eng. 13 (2005) R1.
- [26] L.Z. Cao, J. Shen, N.X. Chen, J. Alloys Compd. 336 (2002) 18.
- [27] J.P. Perdew, K. Burke, Y. Wang, Phys. Rev. B 54 (1996) 16533.
- [28] H.J. Monkhorst, J.D. Pack, Phys. Rev. B 13 (1976) 5188.
- [29] S.N. Mishra, Phys. Rev. B 77 (2008) 224402.
- [30] T. Beuerle, K. Hummler, C. Elsässer, M. Fähnle, Phys. Rev. B 49 (1994) 8802.
- [31] K. Willenborg, R. Zeller, P.H. Dederichs, Europhys. Lett. 18 (1992) 263.
- [32] S. Blügel, Phys. Rev. Lett. 68 (1992) 851.
- [33] S. Stadler, R.P. Winarski, J.M. Maclaren, D.L. Ederer, J. canEk, A. Moewes, M.M. Grush, T.A. Callcott, R.C.C. Perera, J. Electron Spectrosc. Relat. Phenom. 110–111 (2000) 75.
- [34] N.I. Medvedeva, A.L. Ivanovskii, Phys. Solid State 43 (2001) 469.
- [35] D.L. Price, B.R. Cooper, Phys. Rev. B 39 (1989) 4945.
- [36] J.R. Kitchin, J.K. Nørskov, M.A. Barteau, J.G. Chen, Catal. Today 105 (2005) 66.
- [37] B. Xiao, J.D. Xing, J. Feng, C.T. Zhou, Y.F. Li, W. Su, X.J. Xie, Y.H. Cheng, J. Phys. D: Appl. Phys. 42 (2009) 115415.
- [38] W. Zhou, L. Liu, B. Li, P. Wu, Q. Song, Comput. Mater. Sci. 46 (2009) 921.
- [39] S.K.R. Patil, S.V. Khare, B.R. Tuttle, J.K. Bording, S. Kodambaka, Phys. Rev. B 73 (2006) 104118.
- [40] Z.J. Wu, E.J. Zhao, H.P. Xiang, X.F. Hao, X.J. Liu, J. Meng, Phys. Rev. B 76 (2007) 054115.
- [41] D.V. Suetin, I.R. Shein, A.L. Ivanovskii, Physica Status Solidi B 245 (2008) 1590.
- [42] I.R. Shein, D.V. Suetin, A.L. Ivanovskii, Tech. Phys. Lett. 34 (2008) 841.
- [43] M. Lee, R.S. Gilmore, J. Mater. Sci. 17 (1982) 2657.
- [44] T. Sahraoui, A. Kellou, H.I. Faraoun, N. Fenineche, H. Aourag, C. Coddet, Mater. Sci. Eng. B 107 (2004) 1.
- [45] R.R. Reeber, K. Wang, J. Am. Ceram. Soc. 82 (1999) 192.
- [46] V.V. Brazhkin, A.G. Lyapin, R.J. Hemley, Philos. Mag. A 82 (2002) 31.
- [47] M. Nikolussi, S.L. Shang, T. Gressmann, A. Leineweber, E.J. Mittemeijer, Y.Wang, Z.K. Liu, Scripta Mater. 59 (2008) 814.
- [48] J.H. Jang, I.G. Kim, H.K.D.H. Bhadeshia, Comput. Mater. Sci. 44 (2009) 1319.
- [49] D. Music, U. Kreissig, R. Mertens, J.M. Schneider, Phys. Lett. A 326 (2004) 473.
- [50] Y. Yang, H. Lu, C. Yu, J.M. Chen, J. Alloys Compd. 485 (2009) 542.
- [51] J.J. Gilman, B.W. Roberts, J. Appl. Phys. 32 (1961) 1405.
- [52] Y.C. Liang, W.L. Guo, Z. Fang, Acta Physica Sin. 56 (2007) 4847.
- [53] Z.L. Liu, X.R. Chen, Y.L. Wang, Physica B 381 (2006) 139.
- [54] R.C.D. Richardson, Wear 10 (1967) 291.
- [55] A. Šimůnek, J. Vackář, Phys. Rev. Lett. 96 (2006) 085501.
- [56] A. Šimůnek, Phys. Rev. B 75 (2007) 172108.
- [57] D.M. Teter, MRS Bull. 23 (1998) 22.
- [58] F.M. Gao, Phys. Rev. B 73 (2006) 132104.
- [59] F.M. Gao, J.L. He, E.D. Wu, S.M. Liu, D.L. Yu, D.C. Li, S.Y. Zhang, Y.J. Tian, Phys. Rev. B 91 (2003) 015502.
- [60] E. Deligoz, Y.O. Ciftci, P.T. Jochym, K. Colakoglu, Mater. Chem. Phys. 111 (2008) 29.
- [61] G.V. Sin'ko, N.A. Smirnow, J. Phys.: Condens. Matter 14 (2002) 6989.
- [62] O.L. Anderson, J. Phys. Chem. Solids 24 (1963) 909.
- [63] M.K. Drulis, A. Czopnik, H. Drulis, J.E. Spanier, A. Ganguly, M.W. Barsoum, Mater. Sci. Eng. B 119 (2005) 159.
- [64] Y.A. Chang, L.E. Toth, Y.S. Tyan, Metall. Mater. Trans. B2 (1971) 315.